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Cracking analysis on I-II mixed mode crack

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Abstract. Based on the theory of Linear elastic fracture mechanics and the conservation law, the causes of branching of I-II mixed mode crack are researched and analyzed in the view of energy release rate. The impact of parameter f(f=KII/KI), the ratio of stress intensity factor in mode-II crack to stress intensity factor in mode-I crack, on crack branching is first proposed in this paper. Besides, Geometry and mathematical model of I-II mixed mode crack are created. According to the Physical mechanism, crack propagating along the direction of maximum energy release rate, the effective borders on crack branching in crack tip are given in theory. The causes that only enough distance of the main crack propagating can be corresponding with the facts of cracking are described by calculating and analyzing J-integral of I-II mixed mode crack. The research in this paper has practical value in many aspects, such as predicting bifurcating possibility of crack, promoting crack branching and complete development, playing a role in crack arrest and other corresponding engineering fields.

Key words. Conservation law, crack, energy release rate, stress intensity factor.

1. Introduction

In practical engineering structures, most of the cracks are the deformation of mixed mode crack. For example, brittle rock with cracks is a common rock in rock mass engineering, including shale gas exploration, mining and water conservancy. Most of the rock masses are in complex stress condition and usually present rock deformation characteristics of I-II mixed mode crack[1]–[2].However, the propagation direction of the crack under a combined load is not along the direction of original crack plane, but along with a certain angle to the original crack plane. The angle between the original direction and propagation direction of the crack is called crack angle. To mixed mode crack, the direction is just as important as the rate of crack

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propagation, which is an interesting issue in engineering and academic area. Thereas, a large number of theoretical and experimental studies have been carried by many researchers here and abroad[3]. Sma Khan presented a new fracture criterion associated with fractured zone based on regional and plastical deformation characteristics in crack tip. The criterion can reliably predict the crack angle of I-II mixed mode crack and reflects the direct correlation between crack angle and the size of fractured zone[4]. Based on revised shear lag model, Liu Yongguang[5] briefly described the fracture of I-II mixed mode crack by applying the concept of maximum stress concentration factor. By the research, he calculated the crack angle of I-II mixed mode crack which is consistent with the experimental data. According to MCS criterion (maximum circumferential stress criterion), Zhao Yanhua[6] researched propagation of brittle frature under I-II mixed condition, wherein the effect of the non-singular term of T-stress paralleling the cracking direction. The results confirm that the presence of T-stress in crack tip has a significant effect on mode I-II fracture in brittle fracture, especially for mode-II fracture. Shafique[7] and his colleagues analyzed the crack angle of I-II mixed mode crack under different loaded conditions. He considered that the cracking process mainly depends on the direction of resultant force of positive stresses and shear stresses. All these researches above-mentioned promote the development of mixed mode crack in brittle rocks and establish the foundation of theory and experiment for further research.

Thus there are three ways in researching I-II mixed mode cracks in present. They take stress, strains and energy as parameter respectively. The MCS criterion presented by Erodigan and Sih[8] is one of the wildly used theory in researching the propagation direction of fatigue cracks under I-II mixed mode loading. The theory holds that the cracks initially propagate along the direction of maximum value of circumferential stress and the propagation of the cracks result from circumferential stress going up to the critical value.However, Some researchers, such as Tanaka, Abdel-Mageedet. al[9]., have questioned the MSED criterion. They think the MSED criterion lacks theoretical basis in physics. So the papers described the critical crack angle in mode-I crack based on energy release rate and provided cracking angles of mode-I crack in five kinds of branching condition. This paper provides the critical crack angle of I-II mixed mode crack by Loop J-integral and presents the corresponding critical crack angles in different compound ratio KII/ KIin the first time. This method in this paper will simplify this mode of complex problem further and it has good engineering applicability.

2. Integral conservation law

In 1968, Rice and Cherepanov presented the most important concept named J-integral. J-integral not only has the explicit physical significance, but also has the integral path independence. So, it can be an elastic fracture parameter and combines with stress intensity factor in linear elastic fracture mechanics, which become the nub of fracture mechanics.

Assuming a plane problem with perforated crack, it can include plane stress, plane strain and antiplane shear. The stress, strain and displacement field around the crack are expressed$_{ij}$ and$_{i}$ respectively. There are no external force in the crack plane. Selecting a path named around the crack tip, the origin and destination are set in the Lower and upper surface respectively. So the J-integral defined along the path is expressed as follows.

$$J_j = \int_\Gamma (wn_j - T_i u_{i,j})ds \tag{1}$$

where $J_1$and $J_2$are two integral component in eq. (1). When the crack plane is parallel with the axis x_1, $J_1$and J_2 are defined energy release rate of crack propagation and energy release rate of crack opening respectively, , wherein n_1=$cos(n, x)n_2$=$cos(n, y)$. Parameter w, strain energy density, can be obtain by equation w=1/2($_{ij}$ $_{ij}$). T_i is defined stress vector on integral boundary and can be obtain by equation T_i= $_{ij}n_j$.

3. The calculation of I-II mixed mode crack based on J-integral

The stress field of I-II mixed mode crack tip is expressed by the following equation. To elastic body with uniform and isotropic material, the mathematical expression for elastic strain energy density as follows:

$$w = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} = \frac{1+\mu}{2E}[\sigma_{11}^2 + \sigma_{22}^2 + 2\sigma_{12}^2 - \mu(\sigma_{11}^2 + \sigma_{22}^2 + 2\sigma_{11}\sigma_{22})] \tag{2}$$

The equations, wherein the parameter θ is the included angle to the axis x_1 and the unit is rad. The stress field around crack tip of I-II mixed mode crack is substituted in the equation above and the equation will be simplified as follows:

$$w = \frac{1}{2E}\left\{\frac{K_{\rm I}^2}{2\pi r}\left[\frac{1+\mu}{2}\sin^2\theta + (1+\mu)(1-2\mu)(\cos\theta+1)\right] + \frac{K_{\rm II}^2}{2\pi r}\left[-\frac{3(1+\mu)}{2}\sin^2\theta - (1+\mu)(1-2\mu)(\cos\theta-1) + 2(1+\mu)\right] + \frac{K_{\rm I}K_{\rm II}}{2\pi r}\left[4(1+\mu)\sin\frac{\theta}{2}\cos\frac{\theta}{2}(\cos\theta - 1 + 2\mu)\right]\right\} \tag{3}$$

where$\mu$is Poisson ratio and E is modulus of elasticity. The unit of E is GPa. The expressions of the surface stress component are as follows:

$$T_1 = \frac{K_{\rm I}}{\sqrt{2\pi r}}(\frac{3}{4}\cos\frac{3\theta}{2} + \frac{1}{4}\cos\frac{\theta}{2}) + \frac{K_{\rm II}}{\sqrt{2\pi r}}(\frac{5}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\frac{3\theta}{2}) \tag{4}$$

$$T_2 = \frac{K_{\rm I}}{\sqrt{2\pi r}}(\frac{3}{2}\cos\frac{\theta}{2}\sin\theta) + \frac{K_{\rm II}}{\sqrt{2\pi r}}(\frac{3}{4}\cos\frac{3\theta}{2} + \frac{1}{4}\cos\frac{\theta}{2}) \tag{5}$$

The expression of displacement field in I-II mixed mode crack tip as eqs.(5)and

(6).

$$u_1 = \frac{1+\mu}{2E}\sqrt{\frac{r}{2\pi}}\left\{K_{\mathrm{I}}\left[(5-8\mu)\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right]+K_{\mathrm{II}}\left[(9-8\mu)\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right]\right\} \tag{6}$$

$$u_2 = \frac{1+\mu}{2E}\sqrt{\frac{r}{2\pi}}\left\{K_{\mathrm{I}}\left[(7-8\mu)\sin\frac{\theta}{2}-\sin\frac{3\theta}{2}\right]+K_{\mathrm{II}}\left[(8\mu-3)\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right]\right\} \tag{7}$$

Putting eqs. (6)and (7) into and simplifying them, the equations can be obtained as eqs. from(8)to(11).

$$\frac{\partial u_1}{\partial x_1} = \frac{(1+\mu)}{4E}\frac{1}{\sqrt{2\pi r}}\left\{K_{\mathrm{I}}\left[(3-8\mu)\cos\frac{\theta}{2}+\cos\frac{5\theta}{2}\right]+K_{\mathrm{II}}\left[(8\mu-7)\sin\frac{\theta}{2}-\sin\frac{5\theta}{2}\right]\right\} \tag{8}$$

$$\frac{\partial u_2}{\partial x_1} = \frac{(1+\mu)}{4E}\frac{1}{\sqrt{2\pi r}}\left\{K_{\mathrm{I}}\left[(8\mu-9)\sin\frac{\theta}{2}-\sin\frac{5\theta}{2}\right]+K_{\mathrm{II}}\left[(8\mu-5)\cos\frac{\theta}{2}+\cos\frac{5\theta}{2}\right]\right\} \tag{9}$$

$$\frac{\partial u_1}{\partial x_2} = \frac{(1+u)}{4E}\frac{1}{\sqrt{2\pi r}}\left\{K_{\mathrm{I}}\left[7-8\mu\sin\frac{\theta}{2}+\sin\frac{5\theta}{2}\right]+K_{\mathrm{II}}\left[(11-8\mu)\cos\frac{\theta}{2}+\cos\frac{5\theta}{2}\right]\right\} \tag{10}$$

$$\frac{\partial u_2}{\partial x_2} = \frac{(1+\mu)}{4E}\frac{1}{\sqrt{2\pi r}}\left\{K_{\mathrm{I}}\left[(5-8\mu)\cos\frac{\theta}{2}-\cos\frac{5\theta}{2}\right]+K_{\mathrm{II}}\left[(8\mu-1)\sin\frac{\theta}{2}+\sin\frac{5\theta}{2}\right]\right\} \tag{11}$$

Substitute each equations (1) for equation. In the condition of plane strain, J_1-integral and J_2-integral can be expressed respectively as follows.

$$J_1 = \frac{(1-\mu^2)K_{\mathrm{I}}^2}{2\pi E}\int\left[(1-\cos 2\theta)K_{\mathrm{I}}^2+(\cos 2\theta+1)K_{\mathrm{II}}^2+2\sin 2\theta K_{\mathrm{I}}K_{\mathrm{II}}\right]d\theta \tag{12}$$

$$J_2 = \frac{(1-\mu^2)K_{\mathrm{I}}^2}{2\pi E}\int\left[-\sin 2\theta K_{\mathrm{I}}^2+\sin 2\theta K_{\mathrm{II}}^2-2(\cos 2\theta+1)K_{\mathrm{I}}K_{\mathrm{II}}\right]d\theta \tag{13}$$

4. Cracking analysis of I-II mixed mode crack

Coefficient f, the ratio of stress intensity factor in mode-II crack to stress intensity factor in mode-I crack, is introduced into this research. There is the equation f=KII/KI. So, the eqs.(12)and(13)can be revised to eqs.(14) and (15) respectively.

$$J_1 = \frac{(1-\mu^2)K_{\mathrm{I}}^2}{2\pi E}\int\left[(\cos 2\theta+1)f^2+2\sin 2\theta+1-\cos 2\theta\right]d\theta \tag{14}$$

$$J_2 = \frac{(1-\mu^2)K_{\mathrm{I}}^2}{2\pi E}\int\left[(f^2-1)\sin 2\theta-2f\cos 2\theta-2f\right]d\theta \tag{15}$$

The following is crossover point regularities of the integrand which intersect axis θ as shown in the figure from Fig.1to Fig. 4 respectively. the cracking state in the different value of f will be discussed in this paper.

As shown in Fig. 1 and Fig. 3, whatever change on the value of f happens, J_1-integral must be positive value, which mean energy will be released when crack propagation along the direction of x_1. Then the released energy will promote crack propagation further. However, the difference from J_1-integral is that the value of J_2-integral may be not only positive but also negative. Moreover, J_2-integral makes no impression on the cracking. According to the eq.(15), integrand also can be expressed as follows.

$$(f^2-1)\sin 2\theta-2f(1+\cos 2\theta)=2\sqrt{(f^2-1)^2+4f^2}\cos\theta\sin(\theta-\gamma) \tag{16}$$

where $\gamma=\arctan\frac{2f}{f^2-1}$. From Fig. 2 and Fig.4, integral paths of crack tip are shown in Fig. 5 and Fig.6 respectively in the condition of f<1 and f>1.

As shown in Fig. 2, firstly, there will be J_2??s_{12}??s_{34}??<0 when it is in the condition of f<1. It demonstrates that energy need to be absorbed in cracking if the border of crack tip, $s_{12}$and $s_{34,}$ move along the positive direction of axis x_2. However, it is impossible to happen and it can be neglected. Secondly, in the condition of J_2 (s_{23}) >0, it means that energy will be released when the border of crack tip, $s_{23,}$moves along the positive direction of axis x_2. So, the effect of s_{23}on the cracking should be considered. Thirdly, in J_2 (s_{56}, s_{71}) <0, by contrast, when the border of crack tip , $s_{56}$and $s_{71,}$move along the negative direction of axis x_2, the energy will be released. Then $s_{56}$and s_{71}will promote the cracking. Fourthly, when it is in the condition of J_2 (s_{67}) >0, there is no effect on the cracking yet. The analysis on the condition of f>1 is similar to f<1, which will not be discussed in this paper. Especially, in the condition of f=1, there are only two crossover point between J_2-integral and the axis of x_2. Moreover, the Integral are all negative. Therefore, it is shown that the effective border on the cracking are s_{56}, $s_{67}$and s_{71} by adopting the analysis method as above.

It is obvious that there are the same two crossover point, 90deg and 270 deg, between J_2-integral and the axis of x_2. There are different effective area on the cracking as shown in Fig. 2 and Fig.4. Because the size of effective area means the amount of the released energy, so the larger effective area on the cracking, the more

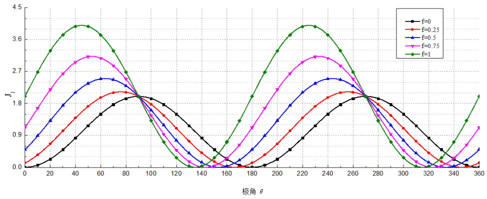

Fig. 1. J_1-integral paths in the condition of f<1

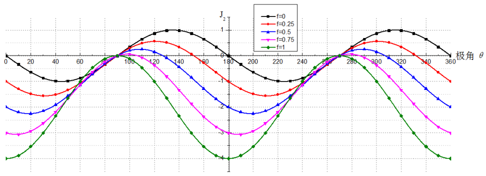

Fig. 2. J_2-integral paths in the condition of f<1

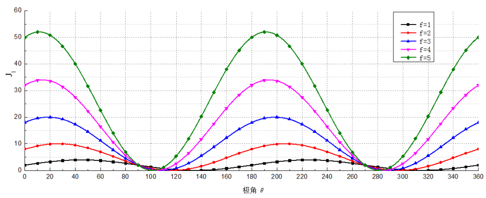

Fig. 3. J_1-integral paths in the condition of f>1

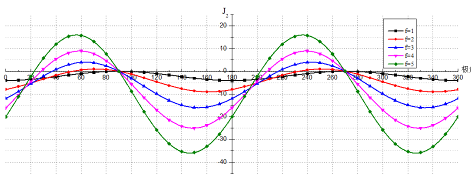

Fig. 4. J_2-integral paths in the condition of f>1

possibilities of the crack branching along this direction. There are two cases in the research. One side, the energy release rate of s_{23}will decrease with the increasing value of f and the energy release rate of s_{56}ands_{71}will increase with the increasing value of f. The other side, when the parameter f is a certain value the energy release rate of J_1-integral along the direction of $x_1$is larger than that of J_2-integral along the direction of x_2. It is just the cause of cracks propagation along the direction of x_1 and making the samples broken. However, if the branching appears in crack propagation, then the stress intensity factor of the crack tip will be decrease and the fracture toughness of the material will be increase.The research on the impact of crack branching by crack border in this paper can promote better branching in the progress of crack propagating. Thereby, the purpose of crack arrest will be realized. Besides, this research provides the basic theory for promoting crack development and improving shield efficiency. It has practical value in engineering.

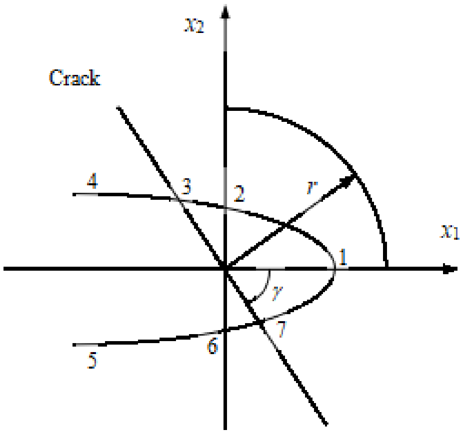

Fig. 5. Integral paths of crack tip in f<1

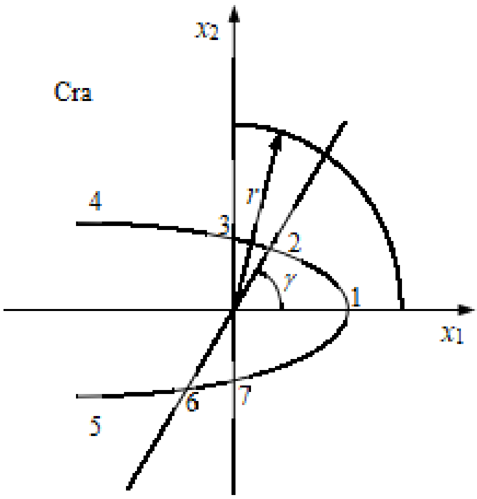

Fig. 6. Integral paths of crack tip in f>1

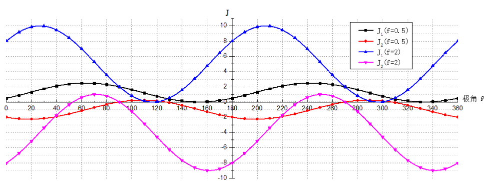

Fig. 7. Integrand distribution of J_1-integral and J_2-integral paths in the condition of f=0.5 and f=2

5. Conclusion

Firstly, geometry and mathematical model of I-II mixed mode crack are created and the model of the energy release rate of infinitesimal border in crack tip, which propagate along the direction of $x_1$and x_2 in the four quadrants, is researched.

Secondly, the mathematical expressions for the energy release rate of I-II mixed mode crack are derived in this paper. By research and analysis on the progress of the cracking in the border based on energy release rate theory , followed basic results are concluded. One side, on the condition of f 1, the effective borders on the crack are s_{23}, $s_{56}$and s_{71}. The other side, on the condition of f=1, it is different. The effective borders on the crack are s_{56}, $s_{67}$and s_{71}. Besides, it is noted that no matter what value the parameter f is, there are two crossover point between J-integral and the axis of θ. J-integral can be J_1-integral or J_2-integral.

Thirdly, the condition of crack branching is that the stress intensity factor of crack tip should increase to the critical value. It is concluded that the energy release rate of J_1-integral along the direction of $x_1$is larger than that of J_2-integral along the direction of x_2 in this paper. Besides, only enough distance the main crack propagating, it can be corresponding with the facts of satisfying bifurcating condition.

Fourthly, cracking will promote more crack tip to be caused and make stress intensity factor of each crack tips decrease dramatically. The degree of cracking is weaken, thereby crack arrest can be realized.

References

[1] N. W. Barry, N. S. Raghu, S. Gexin: *Rock fracture mechanics(principles, design and applications).* Elsevier Science Publishers, Netherlands (1992).

[2] S. Khan, M. Khraisheh: *A new criterion for mixed mode fracture initiation based on the crack tip plastic core region.* International Journal of Plasticty *20* (2004) 55–84.

[3] Y. H. Zhao, J. Chen, H. Zhang: *Influence of T-stress on crack propagation for I-II mixed mode loading.* Engineering Mechanics *27* (2010) 5–11.

[4] M. Shafique, M. Khraisheh: *Analysis of mixed mode crack initiation angles under various loading conditions.* Engineering Fracture Mechanics *67* (2000) 397–419.

[5] F. Erdogan, G. C. Sih: *On the Crack Extension in Plates under Plane Loading and Transverse Shear.* Journal of Basic Engineering *85* (1963) 525–527.

[6] K. Tanaka: *Fatigue Crack Propagation from a Crack Inclined to the Cyclic Tensile Axis.* Engineering Fracture Mechanics *6* (1963) 493–507.

[7] A. Mageed, R. M. Pandey: *Mode Crack Growth under Static And Cyclic Loading in Al-alloy Sheets.* Engineering Fracture Mechanics *40* (1991) 371–385.

[8] Xie. YJ, Hu. XZ, Wang. XH, Cai. M, Wang. W: *Modelling of multiple crack-branching from Mode-I crack-tip in isotropic solids.* Engineering Fracture Mechanics *109* (2013) 105–116.

[9] Xie. YJ, Hu. XZ, Wang. XH, Chen. J, Lee. KY: *A theoretical note on mode-I crack branching and kinking.* Engineering Fracture Mechanics *78* (2011) 919–929.

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